Elementary derivation of the stacking rules of fermionic invertible topological phases in one dimension

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Invertible fermionic topological (IFT) phases are gapped phases of matter with nondegenerate ground states on any closed spatial manifold. When open boundary conditions are imposed, nontrivial IFT phases support gapless boundary degrees of freedom. Distinct IFT phases in one-dimensional space with an internal symmetry group $G_f$ have been characterized by a triplet of indices $((\nu, \rho), \mu)$. Our main result is an elementary derivation of the fermionic stacking rules of one-dimensional IFT phases for any given internal symmetry group $G_f$ from the perspective of the boundary, i.e., we give an explicit operational definition for the boundary representation $((\nu_\wedge, \rho_\wedge), \mu_\wedge)$ obtained from stacking two IFT phases characterized by the triplets of boundary indices $((\nu_1, \rho_1), \mu_1)$ and $((\nu_2, \rho_2), \mu_2)$, respectively.